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Critical Speeds of Turbomachinery: Computer Predictions vs. Experimental Measurements PART II: Effect of Tilt-Pad Bearings and Foundation Dynamics

by

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ABSTRACT

This is the second of two papers describing results of a research project directed at verifying computer programs used to calculate critical speeds of turbomachinery. This part describes measurements made to determine the characteristics of tilt-pad bearings and foundation dynamics. Critical speeds of a 166 kg laboratory rotor on tilt-pad bearings are then compared with predictions from a state-of-the-art damped eigenvalue computer program. Measured natural frequencies of a steam turbine are also compared with computer predictions.

Accuracy of critical speed prediction is shown to depend on accuracy of 1) the "free-free" rotor models, 2) the bearing stiffness and damping coefficients, and 3) the dynamic properties of the foundation, which can be represented by an impedance that must be determined by experimental measurements.

INTRODUCTION

As a result of continuing demand for increased performance, many modern turbomachines for petrochemical and natural gas service are now being designed for operation at shaft speeds approaching the second critical speed. In fact, machines have been purchased and delivered that operated so near a critical speed that difficulty was encountered in maintaining the rotor balance required to ensure acceptable vibration levels. This need for an accurate critical speed prediction capability was further described in the preceding paper [1].

The current paper describes comparisons made between state-of-the-art computer predictions of critical speeds and experimental measurements on turbomachinery rotor/bearing systems of varying complexity. As part of this effort, computer programs were refinements to optimize their accuracy. Experimental techniques were also developed to maximize the repeatability and accuracy of critical speed measurements.

DAMPED CRITICAL SPEED COMPUTER PROGRAM

Damped critical speeds were calculated using a new transfer matrix program. In addition to bearing stiffness and damping, the program included the effects of internal friction, bearing asymmetry, and cross-coupling from bearings, seals, or fluid forces around impellers, as well as foundation impedance effects. The output from the program consists of complex eigenvalues and eigenvectors (mode shapes). Thus, in addition to damped synchronous critical speeds, the program also computes nonsynchronous whirl frequencies with their corresponding logarithmic decrements. The latter are measures of the margin of rotordynamic stability.

The algorithm used was based on a new eigenvalue calculation method set forth in a previous paper by Murphy and Vance [2]. This method requires only one pass through the transfer matrices to generate the characteristic polynomial for the system. A rootfinder subroutine is then used to find the complex eigenvalues. Unlike some of the programs based on the Lund algorithm [3], convergence is always obtained and no critical speeds or whirl frequencies are missed. Computation times are also reduced, usually at least by a factor of one half [2].

The new program was implemented in BASIC on a personal computer. System modeling of the rotor used the same method as other transfer matrix programs. Program refinements described in Part I were incorporated in this program, and the accuracy of the equations was checked using "free-free" natural frequencies as previously covered [1].

EXPERIMENTAL MEASUREMENT OF TILT-PAD BEARING PROPERTIES

Critical speeds of a machine are greatly influenced by the stiffness and damping properties of the bearings which are a function of the operating conditions including shaft speed and bearing load. Tilting-pad bearings are often used on high speed turbomachinery because they favorably influence

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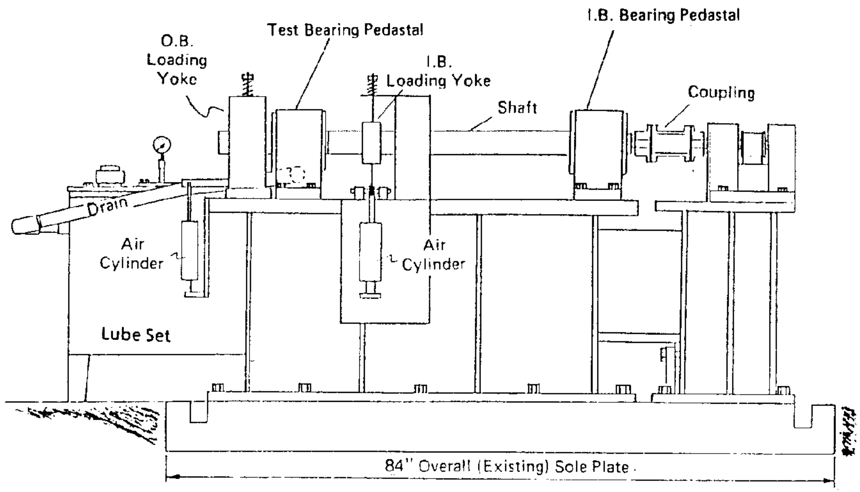

Figure 1. Rotor-Bearing Test Apparatus as Modified for Measuring Fluid-Film Bearing Characteristics

rotordynamic stability. However, only limited experimental work has been published verifying the extensive analytical work that has been published for this type of bearing. Since the two machines selected for this study used tilting-pad bearings, operating characteristics of these bearings were measured in an effort to reduce possible sources of error.

The apparatus used for measuring the bearing characteristics is shown in Figure 1. This apparatus was based on a rotor/bearing test rig, originally built to study vibration of turbomachinery rotors.

Two five-pad, load on pad bearings, with the following specifications, were manufactured specifically for this test:

L = 25.4 mm
D = 63.6 mm
arc length = 60 degrees
radial clearance = 50 μm (.002 in)
preload = 0.0 ±0.1
viscosity = 22.76 cp (3.3 ureyns) @ 100 F (SAE 10)

Details of the apparatus, measurement system, and bearing eccentricity measurements are described in [4].

Vertical Stiffness Measurements

Vertical loads were applied to the test bearing through two pairs of plastic rollers, running on the shaft just inboard and outboard of the bearing. The rollers were mounted in yokes, which were forced down by low friction air cylinders. Figure 2 shows a trace of the shaft center's downward movement (i.e., the static load curve) as the bearing load was slowly increased. The path was smooth with a 10% deviation from vertical. Theoretically, a symmetrical tilting-pad bearing has no cross-coupled stiffness and the path should be straight down. One possible cause of the observed path would be a cross-coupled stiffness equal to 10% of the direct stiffness. Bearing stiffnesses were measured over a range of operating speeds for the bearing load corresponding to the test rotor. Stiffnesses were obtained by dividing small incremental loads applied at each operating condition by the resulting incremental displacements. Table 1 shows typical vertical stiffnesses (K_{yy}) measured. These values agree well with calculated stiffnesses that have been published [5].

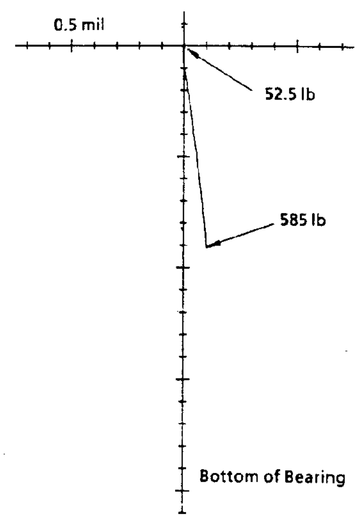

Figure 2. Static Load Curve for Tilting-Pad Bearing
Shaft speed is constant at 3000 rpm.
Bearing load varied from 42.5 lb to 585 lb.

Horizontal Stiffness Measurements

Small horizontal loads were applied by a simple system of weights and pulleys attached directly to the sides of the yokes used to apply the vertical load. For each operating condition (speed-load) small increments in horizontal load were applied using the weights and pulleys, and the resulting incremental deflections were used to calculate horizontal stiffnesses.

As shown in Table 1, the measured horizontal stiffnesses K_{xx} were found to be larger than the measured vertical stiffnesses by as much as a factor of two to three. These large horizontal stiffnesses

Preload is very difficult to measure in small bearings. The actual preload in the tests could have been negative, within the tolerance given.

are contrary to published theoretical results. However, as will be shown later, this rather significant difference in stiffness had a negligible effect on the calculated natural frequencies, since the large bearing damping coefficients [5] dominated the bearing characteristics.

Table 1. MEASURED STIFFNESS VALUES FOR TILT-PAD BEARING

Shaft Speed (rpm)	Bearing Load N (lbs)	K_{xx} (lbs/in)	K_{xy} (lbs/in)	K_{yx} (lbs/in)	K_{yy} (lbs/in)
1500	823(185)	(620000)	(62000)	(-59200)	(592000)
1800	(185)	(606000)	(60600)	(-49700)	(497000)
2200	(185)	(704000)	(70400)	(-34000)	(340000)
4000	(185)	(853000)	(85300)	(-42400)	(424000)
4000	411(92.5)	(679000)	(67900)	(-22400)	(224000)

MACHINE FOUNDATIONS

The critical speeds of the lab rotor-bearing apparatus used in this study were strongly influenced by its concrete and steel foundation and accurate computer simulation included this effect. The computer program used permitted each bearing to be mounted on a foundation mass, which in turn was supported by a spring and damper. For each bearing different spring and damping characteristics could be specified for the horizontal and vertical directions. Different foundation parameters could also be supplied for each bearing, but no direct coupling could exist between the various foundation masses. Numerical estimates for the foundation parameters were obtained from impedance measurements on the foundation when the rotor had been removed.

Measurement of Foundation Parameters

The foundation parameters were obtained from driving point mobility* measurements made directly on the bearing housing. With a measured harmonic force applied to the structure by a shaker, the response of the structure was measured with an accelerometer. (In the measurements discussed here, the accelerometer signal was electronically integrated to give velocity.) The measurement procedure involved driving the shaker with a random noise signal and analyzing the force and acceleration with a two channel frequency analyzer that permitted integration and differentiation of the signals as well as measurement of their transfer functions. Foundation stiffness was obtained from the receptance curve by taking the reciprocal of the value where the frequency approached zero [6]. Foundation damping was obtained from the reciprocal of the peak value of mobility curve. Foundation mass was then calculated using $m = k/\omega^2$, where ω was the measured foundation natural frequency, and k was the stiffness obtained using the system receptance curve.

Natural frequencies of the lab rotor foundation were found using simple rap tests to excite the system. The structure had a horizontal natural frequency of 149 Hz. Vertically, the structure was essentially rigid with a fundamental natural frequency above 300 Hz. Thus, only horizontal measurements were made and just one housing was measured since the two bearing housings were identical. Impedance plots obtained during the foundation tests are shown in Figure 3. The foundation parameters based on these measurements were:

frequency = 149 Hz
stiffness = 36.4×10^6 N/m (208,000 lbf/in)
damping = 9740 N-s/m (55.6 lbf-s/in)
mass = 41.7 kg (92.0 lbm)

The foundation impedance measurements were obtained with the rotor removed from the machine. This eliminated shaft stiffness affecting the stiffness of its supporting structure. The calculated foundation mass was accurate since the rotor mass did significantly alter the measured natural frequency of the foundation. This was verified by comparing the calculated foundation mass to the mass of the rotor. Since the rotor mass was much smaller, it was safely neglected.

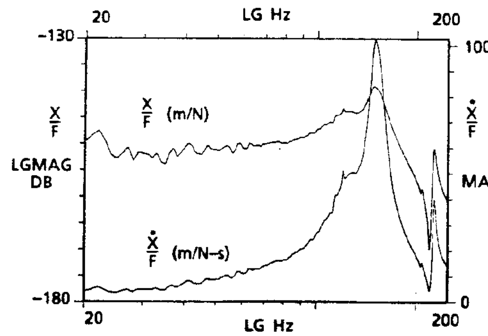

Figure 3. Mechanical Receptance and Mobility Measured On in the Horizontal Directing on the Outboard Foundation of the Rotor-Bearing Test Apparatus

The foundation's characteristics were also measured with the rotor in place and operating at 1000 and 5000 rpm. As before, foundation parameters were obtained by measuring the transfer function of response to excitation. Good results were obtained at 1000 rpm but at 5000 rpm the synchronous vibration of rotor unbalance exciting the foundation was so large the response to the shaker could not be measured. A larger shaker may have provided better results at this higher speed.

CRITICAL SPEED COMPARISONS

Three-Disk Rotor-Bearing System

A variety of methods can be employed to measure the critical speeds of a rotor-bearing system. In industrial machines, the method chosen usually depends upon practical or economic considerations. To eliminate constraints imposed by industrial machines and to control as many parameters as possible, a rotor-bearing test apparatus was built to study vibrational characteristics of a machine.

The rotor was machined from one solid piece of steel and consisted of a 63.5 mm (2.5 in) diameter shaft 1.33 m (52.4 in) long with three large identical disks each 254.0 mm (9.9 in) in diameter and 127 mm (5.0 in) long. The five-shoe tilting pad bearings, previously described, supported each end of the

* Mechanical mobility is defined as the ratio of the measured velocity (magnitude and phase) to the applied force (magnitude and phase). Likewise, receptance is the ratio of the measured displacement to the applied force [6]. If the shaker and velocity transducer are at the same location, the result is called "driving point" mobility.

shaft. Figure 1 in this paper and Figure 4 in [1] show the test rig.

Several different methods were used for measuring the critical speeds of this machine. Critical speeds obtained from coastdown of the rotor were used for comparison with computed values. Damping was measured using the half-power points [7].

The mass-elastic computer model of the rotor shaft given in Table 2 was the same used in [1]. Measured foundation parameters and one set of bearing parameters for 1800 rpm are also shown in this table. Measured bearing stiffnesses from Table 1 were used. Since bearing damping was not measured, calculated values from [5] were used.

Bearing parameters were recalculated at each critical speed for the computed results in Table 3, and at 4000 rpm for Table 4.

Measured and calculated critical speeds of the test apparatus are given in Table 3, and the system mode shapes are shown in Figure 4. The solid lines in this figure represent the calculated shaft mode shapes and the letter "h" represents the calculated foundation displacements at each support. The measured shaft mode shapes are plotted as circles and the measured motion of the test stand is represented as a straight line. Since the third mode was in the vertical plane, the horizontal foundation displacement was not involved and thus is not shown. The coastdown vibration spectrum of the test rig did not show a peak near 140 Hz. The responses to hammer blows on the shaft and housing (Figure 5) also did not show any

TABLE 2. COMPUTER SHAFT MODEL FOR THE THREE-DISK ROTOR-BEARING APPARATUS.

STATION NUMBER	WEIGHT (LP)	LENGTH (IN.)	SHAFT O.D.	SHAFT I.D.	I IN^4	IP LB-IN^2	IT	E*10-6	DISK STIFF IN-LBS/RAD
1	1.320	.753	4.000	0.000	12.57	2.7	1.4	30.00	00.0E+00
2	2.576	1.752	2.501	0.000	1.92	3.6	2.2	30.00	00.0E+00
3	2.436	1.752	2.501	0.000	1.92	1.9	1.6	30.00	00.0E+00
4	2.573	.762	4.000	0.000	12.57	3.7	2.2	30.00	00.0E+00
5	2.605	1.800	2.500	0.000	1.92	3.7	2.2	30.00	00.0E+00
6	2.149	2.021	2.000	0.000	.79	1.4	1.4	30.00	00.0E+00
7	2.232	3.000	2.000	0.000	.79	1.1	1.9	30.00	00.0E+00
8	55.773	4.998	9.900	0.000	471.53	667.6	448.1	30.00	00.0E+00
9	55.347	2.021	2.010	0.000	.80	667.4	447.3	30.00	00.0E+00
10	2.254	3.000	2.010	0.000	.80	1.1	1.9	30.00	00.0E+00
11	55.764	4.996	9.900	0.000	471.53	667.4	447.9	30.00	00.0E+00
12	55.316	2.012	2.005	0.000	.79	667.1	447.1	30.00	00.0E+00
13	2.239	3.000	2.005	0.000	.79	1.1	1.9	30.00	00.0E+00
14	55.845	5.004	9.900	0.000	471.53	668.4	449.0	30.00	00.0E+00
15	55.432	2.075	2.005	0.000	.79	668.2	448.2	30.00	00.0E+00
16	2.267	3.000	2.005	0.000	.79	1.1	1.9	30.00	00.0E+00
17	3.255	2.756	2.500	0.000	1.92	2.2	3.3	30.00	00.0E+00
18	3.130	1.750	2.500	0.000	1.92	2.4	2.7	30.00	00.0E+00
19	2.438	1.760	2.500	0.000	1.92	1.9	1.6	30.00	00.0E+00
20	2.751	2.200	2.500	0.000	1.92	2.1	2.0	30.00	00.0E+00
21	1.641	1.000	1.010	0.000	.05	1.2	1.2	30.00	00.0E+00
22	.229	1.001	1.010	0.000	.05	0.0	0.0	30.00	00.0E+00
23	.111	.001	1.000	0.000	.05	0.0	0.0	30.00	00.0E+00
	------- 369.68	------ 52.414							

INTERNAL FRICTION COEFFICIENT: 0

MATERIAL PROPERTIES ARE THE SAME FOR ALL STATIONS
MASS DENSITY= 0.283 pci
YOUNG'S MODULUS= 30.0E+06 psi
SHEAR MODULUS= 12.0E+06 psi

STATIONS WITH BEARINGS
#3
#19

BEARING DATA

KXX lb/in	KXY	KYX	KYY	CXX lb-s/in	CXY	CYX	CYY
606000	60600	-49700	497000	500	0	0	2100
606000	60600	-49700	497000	500	0	0	2100

FOUNDATION PARAMETERS

	----------HORIZONTAL---------			------------VERTICAL----------		
STATION #	STIFFNESS lb/in	DAMPING lb-s/in	WEIGHT lb	STIFFNESS	DAMPING	WEIGHT
3	208000.0	55.6000	92.0	0.0	0.0000	0.0
19	208000.0	55.6000	92.0	0.0	0.0000	0.0

response near a frequency of 140 Hz. Since the computed log decrement for this mode was the highest of the five calculated, this mode may have been so well damped that the response was not measurable.

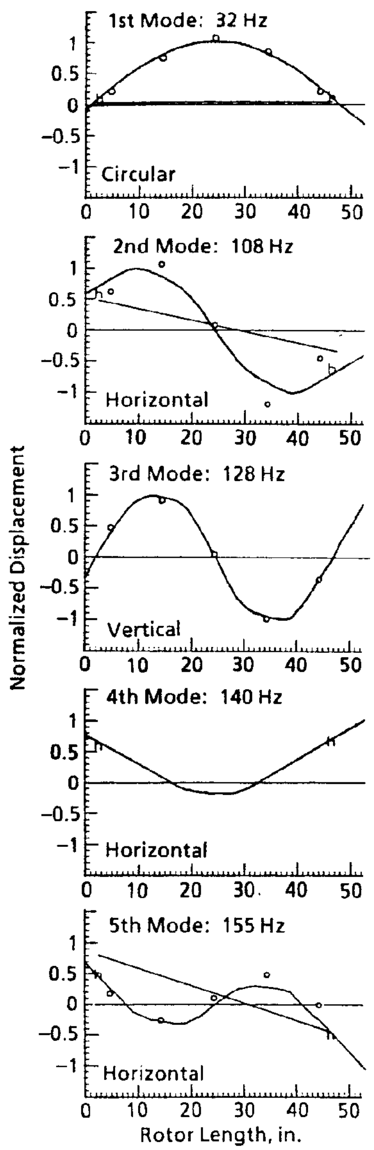

Figure 4. Comparison of Measured and Calculated Mode Shapes of the Three-Mass Rotor-Bearing Test Apparatus

The error in the computed frequency of the 155 Hz mode was probably due to poor modeling of the foundation characteristics. The foundation model used in the program simulated the fundamental natural frequency of the foundation at 149 Hz. The foundation also had a second mode at 225 Hz, which was not considered in the model.

Table 3. MEASURED AND CALCULATED CRITICAL SPEEDS FOR THE 3-DISK LAB ROTOR

Measured frequency Hz	Measured log decrement	Computed frequency Hz	Computed log decrement
30.5	0.11	32.5(6.56%)	0.05
108.6	0.10	103.2(-4.97%)	0.30
125.7	0.10	128.1(1.91%)	0.25
not measured		140.0	0.62
155.0	0.08	169.9(9.61%)	0.40
Overall error		Avg. = 5.76%	

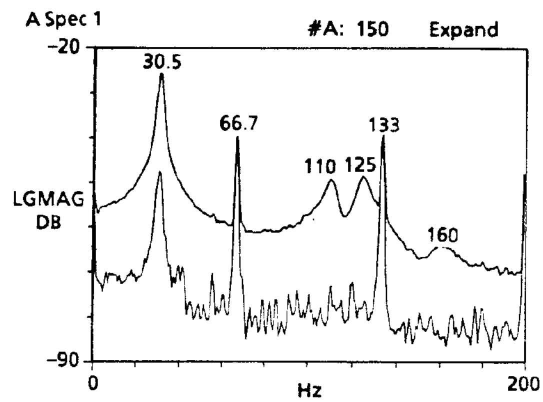

Figure 5. Response of the Experimental Apparatus to Hammer Blows on the Rotor While Rotating at 4000 rpm The lower vibration spectra was the steady state response before the rotor was excited. The top trace resulted from the hammer excitation. The spectra spikes at 66.7 Hz and 133 Hz are at one and two times the rotor operating speed.

As previously mentioned, system natural frequencies were excited by rapping on the shaft and bearing housing with a hammer while the rotor was operating at several constant speeds. Peaks in the frequency analysis of the resulting accelerometer and displacement signals, such as shown in Figure 5, were used to identify the natural frequencies. One typical set of measured natural frequencies and the corresponding calculated natural frequencies for a shaft speed of 4000 rpm are shown in Table 4.

Table 4. RAP TESTS VS. CALCULATED NATURAL FREQUENCIES FOR THREE-DISK LAB ROTOR AT 4000 RPM

Measured Hz	Computed (% error) frequency Hz
31.25	32.7 (4.64%)
110.0	105 (-4.55%)
125.0	128 (2.40%)
157.0	169 (7.64%)
Overall error	Avg. = 4.81%

The lab rotor-bearing apparatus also provided a good example of a backward whirl mode. The frequency spectrum obtained during coastdown through the second, third and fourth critical speeds, showed two spikes at the first critical speed representing subsynchronous whirling. A minor component at 29 Hz represented the backward component, and the major component at 32 Hz represented the forward component and nominal first critical speed. Whirl direction was obtained from the phase of the transfer function of two orthogonal displacement probe signals used to measure shaft vibration. At 32 Hz the signal from the probe mounted at -45° to the vertical axis led the signal from the probe mounted at +45° by 90 degrees indicating forward whirl. At 29 Hz the signal from the probe mounted at -45° lagged by 90 degrees indicating backward whirl at this frequency. The phase angles at the other three critical speeds were either 0 or +180 degrees, indicating these were probably planar modes at the bearings.

Single-Stage Steam Turbine

Extensive measurements were also taken on the single-stage steam turbine that supplied the rotor tested in [1]. This rotor had a single large wheel located near the shaft mid-span. Except for the coupling hub at station 23, the station model used for this rotor is the same as that used in Part I (Table 5). Measured free-free natural frequencies of the rotor without the coupling hub were used to check the rotor model accuracy. Table 6 shows excellent agreement for the first 5 free-free modes. For the

TABLE 5. COMPUTER SHAFT MODEL FOR A SINGLE-STAGE STEAM TURBINE ROTOR.

STATION NUMBER	WEIGHT (LP)	LENGTH (IN.)	SHAFT O.D.	SHAFT I.D.	I IN^4	IP LB-IN^2	IT	E*10-6	DISK STIFF IN-LBS/RAD
1	24.939	2.110	1.998	0.000	.78	.5	.6	30.00	00.0E+00
2	2.782	2.000	1.998	0.000	.78	.9	1.1	30.00	00.0E+00
3	2.036	1.500	2.625	0.000	2.33	1.4	1.2	30.00	00.0E+00
4	2.297	1.500	2.625	0.000	2.33	2.0	1.4	30.00	00.0E+00
5	3.141	2.000	2.994	0.000	3.94	3.2	2.5	30.00	00.0E+00
6	3.985	2.000	2.994	0.000	3.94	4.5	3.6	30.00	00.0E+00
7	5.230	3.250	2.994	0.000	3.94	5.9	6.4	30.00	00.0E+00
8	6.782	2.000	3.993	0.000	12.48	10.7	9.4	30.00	00.0E+00
9	7.088	2.000	3.993	0.000	12.48	14.1	9.4	30.00	00.0E+00
10	11.203	2.750	5.006	0.000	30.83	31.1	21.5	30.00	00.0E+00
11	18.799	4.000	5.006	0.000	30.83	58.9	49.1	30.00	00.0E+00
12	19.687	2.050	6.125	0.000	69.09	75.0	55.3	30.00	00.0E+00
13	89.716	1.000	6.125	0.000	7269.09	4129.4	2068.2	30.00	27.0E+07
14	12.716	2.050	6.125	0.000	69.09	59.6	33.2	30.00	00.0E+00
15	16.067	2.700	5.006	0.000	30.83	63.6	39.4	30.00	00.0E+00
16	15.875	3.000	5.006	0.000	30.83	49.7	35.7	30.00	00.0E+00
17	15.521	4.050	3.990	0.000	12.44	40.4	36.3	30.00	00.0E+00
18	10.256	3.100	2.995	0.000	3.95	17.7	21.1	30.00	00.0E+00
19	5.383	2.300	2.995	0.000	3.95	6.0	6.5	30.00	00.0E+00
20	4.586	2.300	2.995	0.000	3.95	5.1	4.6	30.00	00.0E+00
21	4.064	2.550	2.500	0.000	1.92	4.0	3.9	30.00	00.0E+00
22	3.160	2.000	2.500	0.000	1.92	2.5	2.7	30.00	00.0E+00
23	61.529	.550	1.510	0.000	.26	1.1	1.0	30.00	00.0E+00
24	.140	.001	1.510	0.000	.26	0.0	0.0	30.00	00.0E+00
25	0.000	0.000	1.000	0.000	.05	0.0	0.0	30.00	00.0E+00
	347.02	52.761							

INTERNAL FRICTION COEFFICIENT: 0

MATERIAL PROPERTIES ARE THE SAME FOR ALL STATIONS
MASS DENSITY= 0.283 pci
YOUNG'S MODULUS= 30.0E+06 psi
SHEAR MODULUS= 12.0E+06 psi

STATIONS WITH BEARINGS
#6
#19

BEARING DATA AT 9480 rpm

KXX lb/in	KXY	KYX	KYY	CXX lb-s/in	CXY	CYX	CYY
700000	0	0	350000	1200	0	0	1500
700000	0	0	350000	1200	0	0	1500

damped critical speed calculations, a weight of 267N (60 lbs) was used for the combined effect of the hub and coupling.

Table 6. FREE-FREE NATURAL FREQUENCIES FOR A SINGLE-STAGE STEAM TURBINE ROTOR

Measured Hz	Computed (% error) computed with the damped critical speed program
373	375 (0.54%)
602	600 (-0.3%)
1033	1055 (2.13%)
1262	1240 (-1.74%)
1998	1979 (-0.95%)
Overall error	Avg. = 1.13%

The following properties were used in calculations for the steam turbine five-pad load on pad tilting-pad bearings:

L = 31.75 mm (1.25 inches)
D = 76.2 mm (3 inches)
arch length = 60 degrees
radial clearance = 76.0 μm (3 mils)
preload = 0
bearing load = 693.9 N (156 lbs)

Calculations for this machine were based on the measured bearing stiffnesses discussed earlier in this paper. Damping coefficients were calculated using [5].

Over a decade ago, when this machine was purchased, the manufacturer predicted that all critical speeds were above the maximum operating speed of 11,500 rpm. This apparently was based on a rigid-support analysis performed with no coupling hub, which yields a first critical speed of 13,800 rpm. With the coupling and tilting-pad bearing parameters included, the computed critical speeds provide good agreement with the measured values shown in Figure 6 and Table 7. The logarithmic decrements could not be obtained from the available field measurements.

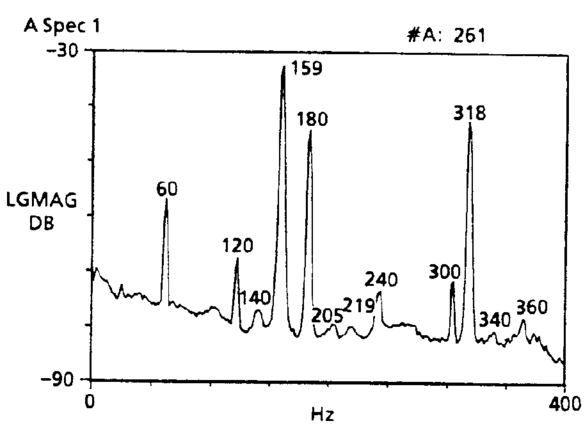

Figure 6. Vibration Spectrum of a Single-Stage Steam Turbine Operating at 9540 rpm (159 Hz)

Table 7. MEASURED AND CALCULATED EIGENVALUES FOR SINGLE-STAGE STEAM TURBINE

Measured frequency Hz	Computed frequency Hz	log decrement	whirl direction
140	142 (1.42%)	.18	backward
158*	150	.17	forward
180**	185	.55	backward
205	201 (1.95%)	.68	forward

*running speed - large amplitude
** possibly the harmonic of 60 Hz electrical noise

The accuracy of the computed critical speeds is excellent in view of the fact that the coupling characteristics were approximated and the bearing parameters were obtained in an indirect manner from laboratory measurements. In this case good results were obtained without considering the machine foundation effects.

Although computed whirl directions are given, the actual whirl directions could not be determined.

IMPORTANCE OF FOUNDATION DYNAMICS AND BEARING ASYMMETRY

Although foundation dynamics were apparently unimportant in the case just presented, the 3-disk lab rotor was a counter example. To show the effect of the foundation, critical speeds were computed without the foundation, but with the tilt-pad bearing asymmetry coefficients retained. Table 8 shows the resulting frequencies, compared with the values measured and the values computed with foundation dynamics included.

Table 8. CRITICAL SPEEDS WITH AND WITHOUT FOUNDATION EFFECTS FOR THE 3-DISK LAB ROTOR

Measured frequency Hz	Computed with foundation Hz	Computed no foundation Hz
30.5	32.5	32.8
108.6	103.2	not predicted
125.7	128.1	139.8
155.0	169.9	not predicted

Omission of the foundation impedance from the computer model eliminates two of the four predicted critical speeds in the operating range.

Many critical speed codes, such as the undamped critical speed program described in [1] incorporate a symmetric model for bearing stiffness ($K=K_{xx}=K_{yy}$). To show the effect of this simplification, critical speeds were calculated using symmetric bearing stiffnesses for both the 3-disk lab rotor (Table 9) and the single-stage steam turbine (Table 10). The symmetric bearing stiffnesses used were the average of the vertical and horizontal values. In the case of the lab rotor, foundation effects were retained.

Tables 9 and 10 show the new computed values of frequency compared to the measured values and the values previously computed using an asymmetric bearing model. It can be seen that, at least for these two machines, a computer model with symmetric bearing stiffness can be used with no sacrifice in critical speed accuracy. Additional parameter sensitivity studies showed that insensitivity of the critical speeds to bearing asymmetry was valid only when the bearings had damping values that dominated the bearing characteristics.

Table 9. CRITICAL SPEEDS WITH AND WITHOUT BEARING ASYMMETRY FOR THE 3-DISK LAB ROTOR

Measured frequency Hz	Computed asymmetric bearings Hz	Computed symmetric bearings Hz
30.5	32.5	32.6
108.6	103.2	102.4
125.7	128.1	128.1
not measured	140.0	137.0
155.0	169.9	168.7

Table 10. EIGENVALUES WITH AND WITHOUT BEARING ASYMMETRY FOR THE SINGLE-STAGE STEAM TURBINE

Measured frequency Hz	Computed asymmetric bearings Hz	Computed symmetric bearings Hz
140	142	141.7
158	150	150.6
180	185	184.8
205	201	199.6

CONCLUSIONS

It is the opinion of the authors that the first two or three critical speeds of industrial machinery can be predicted with errors of less than 7% provided that care is taken to optimize the computer program and to insure accurate values are used for the station model, bearing properties and foundation impedance. A typical rotor/bearing system seems to be well represented by the linear model used in the transfer matrix computer program. The program modifications developed in [1] improve the program calculations. Reliable predictions of critical speeds also depend on the proper choice of bearing and foundation parameters as has been shown here. Dependable methods for accurately estimating these parameters are therefore required.

In many machines bearing damping dominates the bearing characteristics. For these cases the asymmetric bearing stiffnesses in tilting-pad bearings can be replaced with a symmetric model using the average stiffness with little loss of accuracy, even where foundation dynamics are important. However, omission of the foundation impedance from the computer model, in cases where the foundation participates in the modes of interest, can cause some critical speeds to be missed completely.

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