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LABYRINTH SEAL EFFECTS ON ROTOR WHIRL INSTABILITY

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SYNOPSIS The destabilizing effect of labyrinth seals on rotor whirl was first identified by Alford in 1965. Alford's analytical model included the assumption of choked flow at both the inlet and outlet blades of a two blade seal. This paper points to other information which indicates that choked flow can exist only at the exit blade. Under the latter assumption an analysis is performed for a multiblade labyrinth seal. The effects on rotor response and whirl stability are discussed.

INTRODUCTION

1. In recent years many designers of high-performance turbomachinery have been confronted with the problem of self-excited sub-synchronous whirling at a frequency equal to the rotor's first bending critical. In his pioneering paper, Alford (Ref. 1) identifies a possible cause of self-excited whirling to be the varying static pressures within the cavities of a labyrinth seal. In his analysis Alford shows that whenever a rotor is in a whirling configuration the static pressure within the seal varies around the circumference, resulting in a net pressure force acting on the rotor. This pressure variation is due to the compressibility of the gas, as defined by the equation of state for a perfect gas, rather than the gas' inertia or viscosity.

2. To understand how this force is produced consider the leakage flow through a single cavity of a labyrinth seal (see fig. 1) over a small portion of the seal's circumference. In the event that the inlet blade clearance is larger than that of the outlet blade, movement of the rotor toward the stator will cause a greater percentage reduction in flow area at the outlet blade than at the inlet blade. For a short period of time this will create a flow imbalance for the cavity. The inlet flow will exceed the outlet flow since its area has been reduced to a relatively lesser extent. This will cause gas to accumulate within the cavity and so the pressure will rise.

3. The same line of reasoning can be applied to the cavity at a point on the opposite side of the rotor (180°) where the rotor surface is now moving away from the stator. The imbalance of flow caused by the relative changes in flow area will now be tending to evacuate the cavity and so the cavity pressure will be decreasing.

4. It should be noted that a small amount of time is required for the flow imbalance to cause the resulting change in pressure. This causes the pressure variation to lag behind the rotor vibration. The pressure variation created this way is found analytically and integrated around the rotor to obtain the net pressure force acting on the rotor. Due to the afore mentioned time lag this pressure force can be expected to have a tangential as well as a radial component, and thus may be stabilizing or destabilizing depending on seal geometry.

5. The analysis is performed in much the same manner as in Ref. 1. Slightly different assumptions are used and the analysis is performed for a multiblade seal.

NOTATION

$\bar{p}$	steady state, or time average, absolute static pressure
p	time varying portion of absolute static pressure
$\hat{p}$	total absolute static pressure, sum of mean and variance, $\hat{p} = \bar{p} + p$
$\bar{w}$	steady state, or time average, mass flow rate per unit length of circumference
$\hat{w}$	total mass flow rate per unit length of circumference, sum of mean and variance
$\bar{\delta}$	steady state, or time average, clearance, also flow area per unit length of circumference
$\hat{\delta}$	total clearance, sum of mean and variance
x	rotor displacement
C	dimensionless flow coefficient
R	gas constant
T	temperature
F	function of pressure ratio defined by Kearton and Keh (Ref. 2) $\simeq 1$
K,A,B	constants defined in text
P	amplitude of pressure variation
ϕ	phase angle lag of pressure behind displacement
γ	ratio of specific heats
α	$(\gamma-1)/\gamma$

MODEL

6. The flow model contains many of the assumptions used by Alford. The inlet and outlet pressures are assumed to be constant, the flow across each blade to be isentropic and the overall flow to be adiabatic. The rotor motion is assumed to be harmonic* and the resulting changes in seal volume are assumed negligible.

*circular whirling motion is the sum of two harmonic motions in perpendicular directions which are 90 degrees out of phase

Circumferential flow within the seal is neglected and thus so is the effect of shaft rotation. Under these assumptions the results of the analysis apply equally to forward and backward whirling rotors.

7. In the original analysis (Ref. 1) for a two-bladed single cavity seal the flow was assumed choked for both the inlet and outlet blades. Kearton and Keh (Ref. 2) point out, however, that if the flow is assumed adiabatic the flow follows a Fanno line and thus choked flow is possible only at the outlet blade. So for the unchoked flow at the inlet blade (Fig. 2) one can use the relation given by Kearton and Keh (equation 10 in Ref. 2)

$$\hat{w}_1 = C_1\hat{\delta}_1 \sqrt{\frac{F(\bar{P}_o^2 - \hat{P}_1^2)}{RT_o}}$$

By noting that the variational part of the pressure, $\bar{P}_1$, is small compared to P_o and $\bar{P}_1$, and using the binomial expansion, this equation can be rewritten as

$$\hat{w}_1 = C_1\hat{\delta}_1 \sqrt{\frac{F}{RT_o}} \sqrt{\bar{P}_o^2 - P_1^2} \left(1 - \frac{\bar{P}_1}{\bar{P}_o^2 - P_1^2} P_1\right)$$

or

$$\hat{w}_1 = K_1\hat{\delta}_1 \quad (1 - K_1' P_1) \qquad (1)$$

where the constants are funtions only of the steady state flow parameters

$$K_1 = C_1 \sqrt{\frac{F}{RT_o} (\bar{P}_o^2 - \bar{P}_1^2)} = \frac{\bar{w}}{\bar{\delta}_1}$$

$$K_1' = \frac{\bar{P}_1}{\bar{P}_o^2 - \bar{P}_1^2}$$

8. For flow across an intermediate blade one can use the Saint Venant-Wantzel equation written as

$$\hat{w}_n = C_n\sqrt{\frac{2}{\alpha RT_o}} \left(\frac{\hat{P}_n}{\hat{P}_{n-1}}\right)^{1-\alpha} \left(1 - \left(\frac{\hat{P}_n}{\hat{P}_{n-1}}\right)^{\alpha}\right)^{\frac{1}{2}} \hat{\delta}_n\hat{P}_n$$

where

$$\alpha = \frac{\gamma - 1}{\gamma}$$

and γ is the ratio of specific heats. This equation can be rewritten as

$$\hat{w}_n = K_n\hat{\delta}_n\hat{P}_{n-1} \qquad (2a)$$

or

$$\hat{w}_n = K_n'\hat{\delta}_n\hat{P}_{n-1} \qquad (2b)$$

where

$$K_n = \frac{\bar{w}}{\bar{\delta}_n\bar{P}_{n-1}}$$

and

$$K_n' = \frac{\bar{w}}{\bar{\delta}_n\bar{P}_n} \qquad n = 2,3,\ldots,N-1$$

and it is assumed that the pressure ratios are constant

$$\frac{\bar{P}_n}{\bar{P}_{n-1}} = \frac{\hat{P}_n}{\hat{P}_{n-1}}$$

9. For flow across the last blade the flow may or may not be choked. If not choked an expression analogous to equation (1) may be derived.

$$\hat{w}_N = K_N\hat{\delta}_N(1 + K_N' P_{N-1}) \qquad (3a)$$

where

$$K_N = \frac{\bar{w}}{\bar{\delta}_N}$$

and

$$K_N' = \frac{\bar{P}_{N-1}}{\bar{P}_{N-1}^2 - \bar{P}_N^2}$$

Should the flow be choked the flow relation becomes (for air)

$$\hat{w}_N = C_N \frac{0.685}{\sqrt{RT_o}} \hat{\delta}_N\hat{P}_{N-1} = K_c\hat{\delta}_N\hat{P}_{N-1} \qquad (3b)$$

where

$$K_c = \frac{\bar{w}}{\bar{\delta}_N\bar{P}_{N-1}}$$

ANALYSIS

10. By using equations (1), (2) and (3), expressions can be derived for the net rate of accumulation of gas in any cavity in terms of the steady state flow parameters (i.e., the constants K). Writing the total clearance for the nth blade as

$$\hat{\delta}_n = \bar{\delta}_n - x$$

we have for the first cavity

$$\hat{w}_1 - \hat{w}_2 = -A_1P_1 - B_1x \qquad (4a)$$

where

$$A_1 = \frac{P_o^2}{(P_o^2 - \bar{P}_1^2)\bar{P}_1} \bar{w}$$

and

$$B = \frac{\bar{w}}{\bar{\delta}_1} (1 - \frac{\bar{\delta}_1}{\bar{\delta}_2})$$

and second order terms having been neglected. Similarly for an intermediate cavity we have

$$\hat{w}_n - \hat{w}_{n+1} = -A_nP_n - B_nx \qquad (4b)$$

where

$$A_n = 0$$

$$B_n = \frac{\bar{w}}{\bar{\delta}_n} (1 - \frac{\bar{\delta}_n}{\bar{\delta}_{n+1}}) \quad n = 2,3,\ldots,N-2$$

11. It should be noted that the A_n are zero because the pressure ratio across an intermediate blade was assumed constant. For the final cavity when the flow is not choked we have

$$\hat{w}_{N-1} - \hat{w}_N = - A_{N-1}P_{N-1} - B_{N-1}x \qquad (4c)$$

where

$$A_{N-1} = \bar{w} \frac{\bar{P}_N^2}{(\bar{P}_{N-1}^2 - \bar{P}_N^2)\bar{P}_{N-1}}$$

and

$$B_{N-1} = \frac{\bar{w}}{\bar{\delta}_{N-1}} (1 - \frac{\bar{\delta}_{N-1}}{\bar{\delta}_N})$$

Should the flow be choked then the constants become

$$A^c_{N-1} = 0$$

and

$$B^c_{N-1} = \frac{\bar{w}}{\bar{\delta}_{N-1}} \left(1 - \frac{\bar{\delta}_{N-1}}{\bar{\delta}_N}\right)$$

Here the constant A^c_{N-1} is equal to zero because the flow is choked and thus is not affected by the pressure change.

12. The accumulation of gas within the cavity can be further related to the changing pressure by the equation of state for a perfect gas, namely

$$\hat{P}_n V_n = \hat{m}_n R T_n$$

where V_n is the cavity volume per unit length of circumference and m_n is the contained mass per unit length of circumference. Taking the time derivative of this equation with V_n, R and T_n treated as constants we have

$$\dot{\hat{P}}_n = \dot{P}_n = \frac{RT_n}{V_n} \dot{\hat{m}}_n = \frac{RT_n}{V_n} (\hat{w}_n - \hat{w}_{n+1})$$

Combining this equation with one of equations (4) yields an equation of the form

$$\frac{V_n}{RT_n} \dot{P}_n = - A_n P_n - B_n x \quad n = 1,\ldots,N-1$$

13. The rotor motion, x, is assumed harmonic so that

$$x = x_o \sin(\omega t)$$

The resulting differential equation is solved (see Ref. 1) by assuming a solution of the form

$$p_n = p_n \sin(\omega t - \phi)$$

Direct substitution into the differential equation allows one to solve for the two constants as (see Ref. 1)

$$p_n = \frac{-RT_n B_n x_o}{\sqrt{\omega^2 V_n^2 + (RT_n)^2 A_n^2}}$$

$$\tan \phi = \frac{V_n \omega}{RT_n A_n}$$

where ω would be the whirl speed of the rotor system.

14. The pressure force can now be integrated around the circumference of the seal to obtain the net pressure force exerted on the rotor by the gas in the nth cavity (Fig. 3).

$$F_n = \frac{\pi}{2} P_n D_n L_n$$

$$F^r_n = F_n \cos \phi = - \frac{\pi}{2} D_n L_n \frac{(RT_n)^2 A_n B_n}{\omega^2 V_n^2 + (RT_n A_n)^2} x_o$$

$$F^t_n = F_n \sin \phi = - \frac{\pi}{2} D_n L_n \frac{RT_n V_n B_n}{\omega^2 V_n^2 + (RT_n A_n)^2} \omega x_o \qquad (5)$$

D_n is the shaft diameter at the nth cavity and L_n is the distance between the blades.

15. The A_n and B_n are constants which are functions of the steady state flow through the seal. The A_n are always greater than or equal to zero. The B_n are negative if the seal is converging (i.e., clearance is decreasing in the flow direction), positive if the seal is diverging and zero if the clearances are equal. At this point it can be seen that a converging seal will produce a tangential force component in the direction of whirl, and thus will have a destabilizing effect on forward whirl as well as backward whirl. A diverging seal, on the other hand, will always be stabilizing. The tangential force component is directly proportional to the tangential Velocity of the rotor centre and thus can be expressed as an equivalent viscous damper (positive or negative).

16. The radial force component is directly proportional to the rotor displacement and thus act as a radial spring. For a converging seal configuration the radial force is inwards and thus would be a positive spring. A diverging seal would behave as a negative spring since the radial force would be directed outwards.

17. The total effect of the entire seal is obtained by adding the contribution of each individual cavity.

REPRESENTATIVE SEAL CALCULATIONS

18. As a representative case the pressure forces have been calculated for a ten blade labyrinth seal. The choosen seal has an overall length of 171 mm, a height of 4mm and diameter of 200 mm. The overall seal pressure ratio is 10 with an inlet temperature of 361°K and outlet to atmosphere. The blade clearances are 0.15 mm at inlet and increase linearly to 0.60 mm at exit. Since the seal is a diverging one it will produce forces much like a positive viscous damper.

19. For ease of calculation the steady state flow was calculated using Martin's equation, modified by Vermes, for a multiblade labyrinth seal (Ref. 3 and 4), and was found to be 0.224 kg/m-s. Also the pressure ratio across each blade was assumed equal (this assumption is of no consequence for the intermediate cavities). For these conditions the labyrinth seal forces were calculated for a whirl speed and amplitude of 2500 rpm and 0.12 mm respectively. The results are given in Table (1).

20. As can be seen in the table the seal offers very little force in the radial direction. The seal does, however, generate a quite significant force in the tangential direction totalling 2310 N. For the chosen seal configuration this force is a damping force and tranforms into an equivalent viscous damping constant of 88 N-s/mm (500 lb-s/in). This amount of damping is close to that of squeeze film dampers in common use on jet aircraft engines to control synchronous response.

DISCUSSION

21. The resultant pressure force produced within the seal, equation (5), is expressed in terms of the steady state flow rate and cavity pressures. Thus it is permissable for one to measure or calculate the steady flow and pressures by any desirable means for any set of assumptions. In this way one can account for blade geometry effects or perhaps kinetic energy carry-over within a cavity. A suggested method is an iteration scheme which guesses a flow rate and solves for the pressures going from inlet, P_o, to exit, P_N. The correct value for the flow rate will yield the correct value for the outlet pressure (assumed known). As an initial guess for the flow rate one can use experience or perhaps Martin's equation for a multiblade seal. The method of this paper is also easily extendible to stepped seal packings.

22. The most questionable assumption used in the analysis is the omission of circumferential

flow within the seal. Under the action of the circumferential pressure gradient some flow will occur from the region of highest pressure to the region of lowest pressure on the opposite side of the seal. This flow will be resisted by friction and by the inertia of the gas. The regions of high and low pressure are travellingaround the seal at a frequency equal to the whirl speed. Thus the gas at any particular point in the seal is in the high pressure region for part of the whirl cycle and the low pressure region for another part. This will cause the circumferential flow direction to cycle back and forth at a frequency equal to the whirl speed. For sufficiently large frequencies the amplitude of this flow will be small and can be ignored. For low whirl speeds, however, the results of the analysis may not apply. The experimental results obtained by D. V. Wright (Ref. 5) are in contradiction to the predictions of the analysis and a possible explanation is the low whirl speed of his test rotor (about 15 Hertz).

RESULTS

23. One of the most important results of the analysis is that a converging seal can have a destabilizing effect on rotors. This is of particular importance when it is realized that normal wear patterns caused by seal rub will tend to produce this condition on many machines.

24. Another important result is that the use of a diverging seal configuration will create a positive damping effect that would not be present with a seal of constant clearance. Designing the seal in this way enables one to introduce a controllable amount of external damping into a rotor-bearing system. In multistage machines utilizing labyrinth seals between stages one can thus obtain damping at one or more mid-span locations and thus improve the synchronous response, of the rotor. High pressure ring type oil seals are also capable of improving synchronous response, but ring seals used in this manner can have an adverse effect on rotor stability (Ref. 6). The labyrinth seal, on the other hand, will improve the stability characteristics as well as the synchronous response.

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Table 1 Forces for ten blade seal)E SEAL

blade	clearance mm	F_n^r N	F_n^t N
1	0.15	-0.33	-85.5
2	0.20	0	-841.
3	0.25	0	-559.
4	0.30	0	-398.
5	0.35	0	-300.
6	0.40	0	-233.
7	0.45	0	-187.
8	0.50	0	-153.
9	0.55	-0.0008	-0.50
10	0.60	—	—

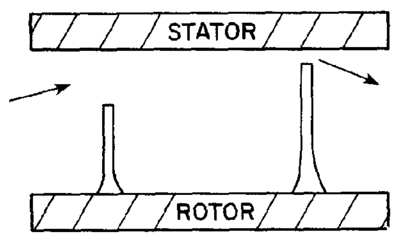

Fig. 1 Single cavity seal

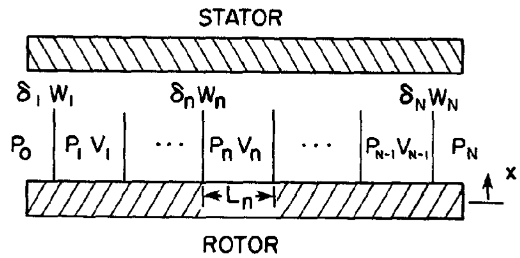

Fig. 2 Multiblade seal notation

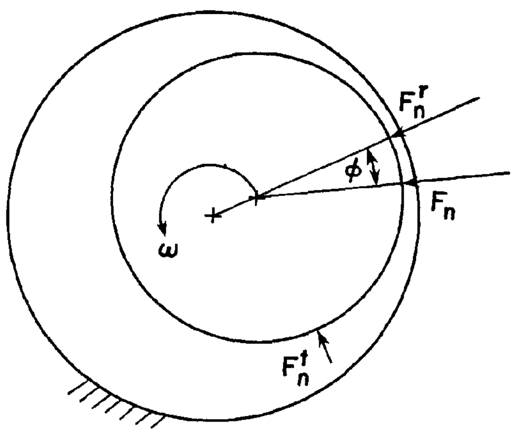

Fig. 3 Phase relationship of forces and motion