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Introduction

The most commonly used tools for rotordynamic analysis employ a transfer matrix representation, based on the works of Myklestad [1] and Prohl [2]. This is a linear analysis in which a rotor is treated as a series of beams. The original procedure, conceived independently by Myklestad and Prohl, is used to determine the undamped synchronous critical speeds of a rotor-bearing system. Subsequently, Lund [3, 4] adapted the procedure for calculating damped unbalance response and damped nonsynchronous natural frequencies and stability of a rotor with asymmetric supports. Lund has also expanded the transfer matrix analysis to handle multiple, interconnected rotors.

For all of these analytical tools, the rotor is represented by a series of uniform (cylindrical) beam elements. In most cases this is perfectly adequate and minor non-uniformities in the rotor (e.g., mild tapers) can be easily and accurately approximated by one or more uniform beam elements. However, there are certain rotor section shapes which are not easily modelled in this manner. Two such rotor section shapes are considered in this paper.

The first of these sections is, in the general case, where the cross-sectional moment of inertia and area vary continually in the axial direction. A set of integral equations are derived which represent the elements of the transfer matrix for this case. For the particular case of a conical section, which is the most frequently encountered non-uniform rotor section, these integral equations are solved in a closed-form for the elements of the transfer matrix. A damped critical speed computer program written by Vance [5], based on the work of Lund [4], is being modified to include this transfer matrix representation for a conical section. An analytical study is being performed to demonstrate the need for and advantages of using this conical section representation. Although the conical section is the only non-uniform rotor section for which an explicit solution is given in this paper, this approach is clearly applicable for any non-uniform rotor section for which these integral equations may be solved (analytically or numerically).

Some rotors have sections which do not behave like beams. In this case, the standard transfer matrix representation is totally inadequate. The most common rotor section of this type is the trunnion. A trunnion is essentially a circular plate, sup-
where \( \theta \) is a function of \( z \) only. The bending moment, \( M \), at any point along the shaft section may be represented in terms of the bending moment and shear force at \( z = 0 \) (\( M_s \) and \( V_s \)), as shown in equation (2)

\[
M = M_s + V_s \theta
\]

Rearranging the terms in equation (1), substituting equation (2) and integrating, as shown in equation (3), results in equation (4)

\[
\int \theta \, dz = \frac{M_s}{E_I} \int_0^z \frac{V_s}{E_I} \, dz + \frac{V_s z^3}{E_I 2} + \theta_0
\]

where \( \theta_1 \) is a constant of integration. By solving equation (4) for \( \theta \) at \( z = 0 \) (which is known to be \( \theta_0 \)), \( C_1 \) is determined to be equal to \( \theta_0 \). Consequently, \( \theta \) is given by equation (5) and \( \theta_1 \) (\( \theta \) at \( z=l \)) is given by equation (6)

\[
\theta = \frac{M_s}{E_I} + \frac{V_s z^3}{E_I 2} + \theta_0
\]

\[
\theta_1 = \frac{M_s}{E_I} + \frac{V_s}{2E_I} + \theta_0
\]

where \( l \) is the length of the shaft section.

Including shear deformation, the rotation \( \theta \) of the shaft section is related to the displacement \( x \) by

\[
\theta = \frac{V_s + dx}{\alpha GA} \frac{dz}{dx}
\]

where \( G \) (shear modulus), \( \alpha \) (cross-sectional shape factor), and \( A \) (cross-sectional area) are invariant for a uniform shaft section, and \( z \) is a function of \( x \) only. Substituting equation (7) into equation (5) and rearranging terms gives

\[
\frac{dz}{dx} = \frac{M_s}{E_I} \frac{x}{E_I 6} + \frac{V_s}{E_I 6} + \theta_0 - \frac{V_s}{\alpha GA}
\]

Multiplying through by \( dx \) and integrating both sides yields

\[
x - \theta_2 = \frac{M_s}{E_I 2} + \frac{V_s z^3}{E_I 6} + \frac{\theta_0}{\alpha GA} z + x_0
\]

Letting \( z \) equal to zero in equation (9) reveals that \( C_0 \) (a constant of integration) must be equal to \( x_0 \). Thus, \( z \) is given by equation (10) and \( x_0 \) is given by equation (11)

\[
x = \frac{M_s}{E_I 2} + \frac{V_s}{E_I 6} + \frac{\theta_0}{\alpha GA} z + x_0
\]

\[
x_0 = \frac{M_s}{E_I 2} + \frac{V_s}{E_I 6} + \frac{\theta_0}{\alpha GA} z + x_0
\]

From equation (2), \( M_s \) may be represented by

\[
M_s = M_0 + V_0 \theta
\]

and \( V_0 \) is merely equal to \( V_0 \). The standard transfer matrix may then be assembled from equations (6), (11) and (12) to give

\[
\begin{bmatrix}
    x_1 \\
    \theta_1 \\
    M_1 \\
    V_1
\end{bmatrix} =
\begin{bmatrix}
    1 & l & \frac{V_s}{2E_I} & \left( \frac{V_s}{6E_I} - \frac{l}{\alpha GA} \right) \\
    0 & 1 & l & \left( \frac{M_s}{2E_I} \right) \\
    0 & 0 & 1 & \frac{M_0}{E_I} \\
    0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    x_0 \\
    \theta_0 \\
    M_0 \\
    V_0
\end{bmatrix}
\]

\[
\begin{pmatrix}
    \frac{dz}{dx} \\
    \frac{d\theta}{dz}
\end{pmatrix} =
\begin{pmatrix}
    1 & \frac{E_I}{2} & \left( \frac{E_I}{6} - \frac{l}{\alpha GA} \right) \\
    0 & 1 & l & \left( \frac{M_s}{2E_I} \right) \\
    0 & 0 & 1 & \frac{M_0}{E_I} \\
    0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
    x_0 \\
    \theta_0 \\
    M_0 \\
    V_0
\end{pmatrix}
\]
M = LUMPED MASS AT LEFT END OF SHAFT SECTION
I_T = LUMPED TRANSVERSE MASS MOMENT OF INERTIA AT LEFT END OF SHAFT SECTION
I_P = LUMPED POLAR MASS MOMENT OF INERTIA AT LEFT END OF SHAFT SECTION

\[ M = E(z)I(z) \frac{d\theta}{dz} \] (14)

where the modulus of elasticity, \( E \), and the cross-sectional moment of inertia, \( I \), are functions of \( z \). The bending moment, \( M \), may still be represented by equation (2). However, equations (3) and (4) are replaced by equations (15) and (16)

\[ \int d\theta = M_0 \int_a^b \frac{dz}{E(z)I(z)} + V_o \int_a^b \frac{zd\theta}{E(z)I(z)} \] (15)

\[ \theta - C_1 = M_0 \int_a^b \frac{dz}{E(z)I(z)} + V_o \int_a^b \frac{zd\theta}{E(z)I(z)} \] (16)

where the values of \( E \) and \( I \) at the left end of the shaft section (see Fig. 2) are \( E(a) \) and \( I(a) \), respectively. The general value \( a \) is used to represent the left end of the shaft section, instead of zero, to ensure that the integrands are defined throughout the interval of integration. \( M_o \), \( V_o \), \( x_o \), and \( \theta_o \) are the respective values at \( z = a \).

For \( z \) equal to \( a \), the integrals in equation (16) are equal to zero and \( C_1 \) (a constant of integration) must be equal to \( x_o \). Then, \( \theta \) and \( \theta_o \) may be given by equations (17) and (18), respectively.

\[ \theta = M_0 \int_a^b \frac{dz}{E(z)I(z)} + V_o \int_a^b \frac{zd\theta}{E(z)I(z)} + \theta_o \] (17)

\[ \theta_1 = M_0 \int_a^b \frac{dz}{E(z)I(z)} + V_o \int_a^b \frac{zd\theta}{E(z)I(z)} + \theta_o \] (18)

Once expressions for \( E(z) \) and \( I(z) \) have been specified, the integrals in equation (18) may be evaluated either analytically or numerically.

For a non-uniform shaft, equation (7) is replaced by

\[ \theta = \frac{V_o}{\alpha(z)G(z)A(z)} + \frac{dx}{dz} \] (19)

where the cross-sectional shape factor, \( \alpha \), the shear modulus, \( G \), and the cross-sectional area, \( A \), are functions of \( z \). Substituting equation (19) into equation (17) and rearranging terms gives

\[ dx = M_0 \int_a^b \frac{dz}{E(z)I(z)} + V_o \int_a^b \frac{zd\theta}{E(z)I(z)} \]
integrals in equations (25-28). The error introduced as a result of this assumption is shown to be generally very small. The expression for the cross-sectional area, \( A(z) \), of the conical section is exact for any general conical section (including the case in which the inside and outside surfaces are not parallel). Thus, the evaluation of the integral for the shear term in equation (27) is exact. The expressions presented below for the mass and transverse and polar mass-moments of inertia of a conical section are also exact for any general conical section.

The cross-sectional moment of inertia, \( I(z) \), for a hollow circular section is given by

\[
I(z) = \frac{\pi}{64} (D(z) - D(z))
\]

\[
= \pi \frac{r(z)}{r(z)^3} \left[ 1 + \frac{t(z)}{r(z)} \right]^2 \tag{29}
\]

where \( D(z) \) is the outside diameter at \( z \); \( D(z) \) is the inside diameter at \( z \); \( r(z) \) is the average radius at \( z \); and \( t(z) \) is the wall thickness at \( z \). These parameters, shown in Fig. 3, are related by

\[
r(z) = \frac{1}{4}(D(z) + D(z)) \tag{30}
\]

Fig. 3 Model of Conical Section for Transfer Matrix Analysis

\[
t(z) = \frac{1}{4}(D(z) - D(z)) \tag{31}
\]

Assuming that \( t \) is "small" compared to \( r \), and that \( t \) is not a function of \( z \), equation (29) reduces to

\[
I(z) = \pi \frac{r(z)}{r(z)^3} \tag{32}
\]

which is equivalent to the typical "thin-walled" approximation for the cross-sectional inertia of tubes. The error induced by the thin-walled approximation is given by

\[
\text{% Error} = \frac{1}{4} \left( \frac{t}{r(z)} \right)^2 \times 100 \tag{33}
\]

From equation (33), it can be seen that this error for the case of a wall thickness equal to one-third of the average radius (not a very thin wall) is less than three percent. In most practical cases, the wall thickness (and thus the induced error) is substantially less than this. If desired, the expressions obtained from equations (25-28) using this approximation could be modified by "fudge" factors to compensate for this error. However, as these factors would have to be functions of \( t/r(z) \) and these particular errors are generally very small, the addition of these factors is not warranted.

For the case of a conical section, \( r(z) \) is given by the function

\[
r(z) = r_0 + (r_1 - r_0)z/l = r_0(1 + pz/l) \tag{34}
\]

where \( r_0 \) is the average radius (average of inside and outside radii) at the "left" end of the shaft section; \( r_1 \) is the average radius at the "right" end of the shaft section; \( l \) is the length of the shaft section; the analysis of the shaft section is performed from left to right; and \( \rho \) is a dimensionless constant defined by

\[
\rho = (r_1 - r_0)/r_0 \tag{35}
\]

Substituting equation (34) into equation (32) results in an expression for cross-sectional area moment of inertia for a conical section given by

\[
I(z) = I_0(1 + \rho z/l)^3 \tag{36}
\]

where the wall thickness, \( t \), is the average value for the shaft section and

\[
I_0 = \pi r_0^4 \tag{37}
\]

For this conical section analysis, the values of \( E \), \( G \) and \( \alpha \) are assumed to be constant. Thus, \( I \) and \( A \) become the only parameters in equations (25-28) which remain a function of \( z \). Substituting equation (36) into equation (25) results in the expression

\[
k_1 = \frac{1}{E I_0} \int_0^1 \frac{dz}{(1 + \rho z/l)^3} \tag{38}
\]

where \( a \) is again set equal to zero. Performing the integration in equation (38) gives

\[
k_1 = \left( \frac{l}{2EI_0} \right) \left( 1 + \frac{\rho}{1 + \rho} \right)^3 \tag{39}
\]

Substituting equation (36) into equation (26) results in

\[
k_2 = \left( \frac{B}{2EI_0} \right) \left( \frac{1}{1 + \rho} \right)^3 \tag{40}
\]

Substituting equation (36) into equation (28) gives

\[
k_4 = \left( \frac{B}{2EI_0} \right) \left( \frac{1}{1 + \rho} \right)^3 \tag{41}
\]

Performing the inside integration gives

\[
k_4 = \left( \frac{B}{2EI_0} \right) \left( \frac{1}{1 + \rho} \right)^3 \tag{42}
\]

Evaluating the outside integral gives

\[
k_4 = \left( \frac{B}{2EI_0} \right) \left( \frac{1}{1 + \rho} \right)^3 \tag{43}
\]

The two integrals in equation (27) may be evaluated separately. These integrals are referred to as \( k_5 \) and \( k_6 \) such that

\[
k_5 = \frac{1}{E} \int_0^1 \int_0^l \frac{dz dz'}{I(z)} \tag{44}
\]

\[
k_6 = \frac{1}{E G} \int_0^l \frac{dz}{A(z)} \tag{45}
\]

Substituting equation (36) into equation (44) results in

\[
k_5 = \left( \frac{B}{2EI_0} \right) \left( \frac{1}{1 + \rho} \right)^3 \tag{46}
\]

Performing the inside integration gives
The mass for the left half of the conical section, \( m_l \), is given by

\[
m_l = \delta \int_0^{b_0} \int_{a_l(0)}^{a_l(\pi/2)} \int_0^r \sigma \, d\theta \, dr \, dz \]

where \( \delta \) is the density of the shaft material, \( a(z) \) and \( b(z) \) are given by

\[
a(z) = r(z) - t(z)/2 \]

and \( b(z) = r(z) + t(z)/2 \)

and \( r(z) \) and \( t(z) \) are given in equations (34) and (52), respectively. Evaluation of the integrals in equation (59) results in the following expression for the mass of the left half of the conical section,

\[
m_l = m_0[1 + 1/4(\rho + \tau) + 1/12(\rho \tau)] \]

where \( \rho \) and \( \tau \) are defined in equations (35) and (53), respectively, and \( m_0 \) is given by

\[
m_0 = \pi r_0^2 b_0 \]

Equations (62) and (63) do not depend on any simplifying assumptions.

It should be noted that the values of \( \rho \), \( \tau \), \( r_0 \) and \( t_0 \) used for the mass and inertia calculations for the conical section need not be the same as those used for the shaft flexibility calculations (although in most cases they are the same). For example, in the case of a built-up shaft section composed of one cone inside another cone, it may be desirable to consider only the inside cone for the shaft flexibility analyses while considering the composite of both cones for the mass and inertia analyses.

The mass of the right half of the conical section, \( m_r \), is found in a similar manner as above from

\[
m_r = \delta \int_0^{b_0} \int_{a_l(0)}^{a_l(\pi/2)} \int_0^r \sigma \, d\theta \, dr \, dz \]

The integrals in equation (64) are evaluated to give

\[
m_r = m_0 \left[ 1 + \frac{3}{4}(\rho + \tau) + \frac{7}{12}(\rho \tau) \right] \]

The polar moment of inertia of the left half of the conical section, \( I_{P1} \), is obtained from the expression

\[
I_{P1} = \pi r_0^4 \int_0^{b_0} \int_{a_l(0)}^{a_l(\pi/2)} \int_0^r \rho \, d\theta \, dr \, dz \]

The integrals in equation (66) are evaluated to give

\[
I_{P1} = I_{P0}[\pi \delta \{5(4+\rho) (8+4\rho + \rho^2) + \tau(40+40\rho + 15\rho^2 + \rho^3)\] + \frac{\tau^2}{4}\{5(4+\tau) (8+4\tau + \tau^2) + \rho(40+40\tau + 15\tau^2 + \tau^3)\}] \]

where \( I_{P0} \) is defined as

\[
I_{P0} = (m_0 r_0^2) / 160 \]

Similarly, the polar moment of inertia of the right half of the conical section, \( I_{P1} \), is derived from

\[
I_{P1} = \pi r_0^4 \int_0^{b_0} \int_{a_l(0)}^{a_l(\pi/2)} \int_0^r \rho \, d\theta \, dr \, dz \]

Evaluation of the integrals in equation (69) results in...
The transverse moment of inertia of the left half of the conical section, $I_{T1}$, is obtained from the expression

$$ I_{T1} = \int (r^2 \cos \theta + s^2) dm $$

$$ = \delta \int_0^{\pi/2} \int_0^{1/2} \int_0^{*} (r^2 \cos \theta + s^2) r dr dz ds $$

After evaluating the inside integral, equation (71) can be rearranged to give

$$ I_{T1} = \frac{I_{PL}}{2} + 2\pi \delta \int_0^{\pi/2} \int_0^{1/2} r^2 dr dz $$

The remaining integrals are evaluated to give

$$ I_{T1} = \frac{I_{PL}}{2} + \frac{m_o B}{480} \left[ 40 + 15(\rho + \tau) + 6\rho \tau \right] $$

Similarly, the transverse moment of inertia for the right half of the conical section is derived from

$$ I_{T2} = \int (r^2 \cos \theta + (1-\varepsilon)^2) dm $$

$$ = \delta \int_0^{\pi/2} \int_0^{1/2} \int_0^{*} (r^2 \cos \theta + (1-\varepsilon)^2) r dr dz ds $$

The integrals in equation (74) may be evaluated to obtain

$$ I_{T2} = \frac{I_{PR}}{2} + \frac{m_o B}{2400} \left[ 200 + 125(\rho + \tau) + 8\rho \tau \right] $$

Clearly, if either $\rho$ or $\tau$ (or both) is equal to zero, equation (82), (85), (67), (70), (73) and (75) reduce to simpler forms. In fact, if both $\rho$ and $\tau$ are equal to zero, these equations reduce to the corresponding expressions for a cylindrical shaft section. Note that the transverse moment of inertia for each half of the conical section is taken about an axis through the corresponding end of the shaft section, which is consistent with standard transfer matrix analysis procedures.

For a torsional, or coupled torsional-lateral transfer matrix analysis, a rotor model with a conical section is necessary only to perform a flexibility analysis similar to that described above, for torsional flexibility. The polar mass moment of inertia is given above in equations (67) and (70)). The cross-sectional polar moment of inertia, $I(z)$, is equal to the transverse cross-sectional moment of inertia, $I(z)$. In this case, $\phi_0$, the angle of twist at the right end of the shaft section is given by

$$ \phi_1 = T_0 \int_{a}^{*} \int_0^{*} F(z)G(z) + \phi_0 $$

where $T_0$ and $\phi_0$ are the torque and angle of twist at the left end of the shaft section, respectively. For a shaft section with constant values of $G$ and $E$, equation (76) may be rewritten as

$$ \phi_1 = 2T_0 \int_{a}^{*} F(z)G(z) $$

where $k_3$ is defined in equation (25) for a general shaft section and in equation (91) for a conical shaft section.

Thus, by the use of the appropriate equations, as defined above, the modelling of a conical section may be handled directly in a transfer matrix rotordynamic analysis.
\[ M_{rt} = (1-\mu) \frac{D}{\omega^2} \left( \frac{1}{\rho} \frac{\partial W}{\partial \theta} - \frac{1}{\rho^2} \frac{\partial W}{\partial \theta} \right) \]  
(85)

\[ Q_r = -\frac{D}{\omega^2} \frac{\partial}{\partial \theta} \left( \frac{\partial W}{\partial \phi} + \frac{1}{\rho} \frac{\partial W}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial W}{\partial \theta} \right) \]  
(86)

where \( \mu \) is Poisson's ratio and \( D \) is the “plate stiffness” given by

\[ D = \frac{E h^3}{12(1-\mu^2)} \]  
(87)

and \( E \) is the modulus of elasticity. Equation (82) is based on the assumption that the trunnion sidewall is built-in to the smaller adjacent shaft section.

One additional boundary condition is required, which depends on the edge support. For the case of a pinned edge support, this boundary condition is

\[ (M_r)_{\rho=1} = 0 \]  
(88)

Substituting equation (79) into equations (81), (82), (83) and (88) results in four equations which may be solved for the coefficients, \( A, B, C, \) and \( F \), from equation (79). The resulting expression for \( W \) is [6]

\[ W = \frac{-M_0}{8\pi D[3(\mu+1)-(1-\mu)\beta]} \left[ -[(1+\mu) + (1-\mu)\beta] \rho^2 
+ (1+\mu) (1-\beta^2) \rho + 2[3(\mu+1) - (1-\mu)\beta] \rho \right] \rho \cos \theta \]  
(89)

The angular flexibility, \( k_A \), is given by

\[ k_A = \frac{\psi}{M} \]  
(90)

where \( \psi \), the angle between the two adjacent shaft sections, is derived from

\[ \psi = \left( \frac{\partial W}{\partial \phi} \right)_{\rho=\beta} = \frac{1}{\alpha} \left( \frac{\partial W}{\partial \rho} \right)_{\rho=\beta} = \frac{1}{\alpha} \left( \frac{W}{\rho} \right)_{\rho=\beta} \]  
(91)

Substituting equations (87) and (89) into equation (91), and the result into equation (90) gives

\[ k_A = \frac{3(1-\mu)^2}{\pi E \rho^4} \left[ \frac{\left[ (\mu-1) \right] + 2(\mu+1) \rho}{(\rho+1) - \ln \beta} \right] \]  
(92)

For the case of a built-in edge support, the fourth boundary condition is

\[ \left( \frac{\partial W}{\partial \rho} \right)_{\rho=1} = 0 \]  
(93)

Substituting equation (93) for equation (88) and performing the procedure described above, results in the following expressions for \( W \) and \( k_A \)

\[ W = \frac{M_0}{8\pi D[ \beta(\beta+1) - \beta^2 \rho + 2(\beta+1) \rho \ln \rho] \cos \theta} \]  
(94)

\[ k_A = \frac{3(1-\mu)^2}{\pi E \beta^2} \left[ \frac{(\beta-1)}{(\beta+1) - \ln \beta} \right] \]  
(95)

The form in which the trunnion model is included in the transfer matrix analysis is dependent on the orientation of the trunnion in the rotor being analyzed. The angular orientation of the shaft at the right end of the trunnion section (analyzing the rotor from left to right) is

\[ \theta_1 = \theta_0 + k_A \rho \]  
(96)

where \( \rho \) represents the moment at the small diameter end of the trunnion. If the left end of the trunnion is the small diameter end, \( M \) is equivalent to \( M \), and

\[ \theta_1 = \theta_0 + k_A \rho \]  
(97)

Thus, equation (24) is modified such that

\[ k_1 = k_A \]  
(98 and

\[ k_3 = k_A = k_4 = 0 \]  
(99)

However, if the right end of the trunnion is the small diameter end, \( M \) is equivalent to \( M \), and

\[ \theta_1 = \theta_0 + k_1 \rho + k_2 \]  
(100)

from equation (12). In this case, equation (24) is modified with the substitutions

\[ k_1 = k_2 \]  
(101)

and

\[ k_3 = k_4 = 0 \]  
(102)

However, since \( l \) for a trunnion is generally very small, \( k_2 \) in equation (102) may be approximated as being equal to zero. Under this assumption, the orientation of the trunnion does not make any difference.

Thus, a realistic model for a trunnion can be directly represented in a transfer matrix analysis. The mass and polar and transverse moments of inertia for the trunnion sidewall (which are generally very small) are calculated using the equations for a solid cylinder which are used in standard transfer matrix modelling procedures.

Summary and Conclusions

Ordinary transfer matrix analysis modelling procedures do not adequately account for certain unusual rotor section shapes. Two of these shapes which occur frequently are conical sections and trunnions. Direct representation of conical sections and trunnions for transfer matrix analyses provide for increased accuracy and simplicity of the transfer matrix modelling procedures. A framework is also presented upon which an analysis similar to the conical section analysis can be built to take into account any other non-uniform shaft section which may be of interest. This procedure is extended to include the parameters required for torsional, or coupled torsional-lateral, transfer matrix analysis.

The analysis presented for a trunnion is based on a theory of elasticity approach for a circular disk supported at its edge. Since a beam analysis is inappropriate for a trunnion, a standard transfer matrix model of a trunnion is a very inaccurate representation. Consequently, in order to get a reasonable representation of a trunnion, it is necessary to use a modelling procedure such as that presented herein, which takes into account the true mode of deformation of the trunnion.

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References


